Some Applications of Trigonometry

Exercise 5.1

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) A pole of 6 m high casts a shadow $2\sqrt{3}$ m long, then sun's elevation is

(*a*) 60° (*b*) 45° (*c*) 30° (*d*) 90°

(2) At some time of the day, the length of the shadow of a tower is equal to height. Then the sun's altitude at that time is

(*a*) 30° (*b*) 60° (*c*) 90° (*d*) 45°

(3) The angle of depression of a car standing on the ground, from the top of a 75 m high tower is 30°. The distance of the car from the base of the tower (in m) is:

(*a*) $25\sqrt{3}$ (*b*) $50\sqrt{3}$ (*c*) $75\sqrt{3}$ (*d*) 150

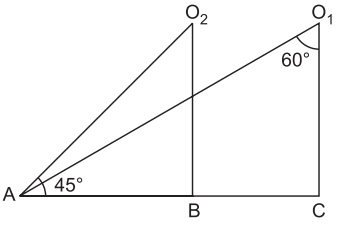
(4) In the given figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A respectively are

(*a*) 30°, 45°

(*b*) 45°, 60°

(*c*) 60°, 75°

(*d*) 60°, 30°



2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

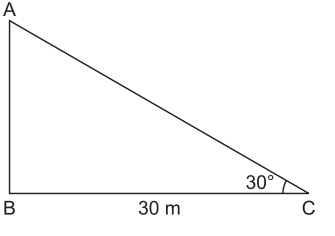
(1) **Assertion (A):** If the angle of elevation of Sun, above a perpendicular line (tower) decreases, then the shadow of tower increases.
Reason (R): It is due to decrease in slope of the line of sight.

(2) **Assertion (A):** When we move towards the object, angle of elevation decreases.
Reason (R): As we move towards the object, it subtends large angle at our eye than before.

3. Answer the following:

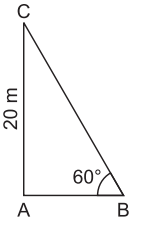
(1) A kite is attached to a string. Assuming that there is no slack in the string, find the height of the kite above the level of the ground, if the length of the string is 54 m and it makes an angle of 30° with the ground. **[Imp.]**

(2) In the given figure, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower. [CBSE Standard 2020]



(3) The ratio of the length of a vertical rod and the length of its shadow is 1 : $\sqrt{3}$. Find the angle of elevation of the sun at that moment? [CBSE Standard 2020]

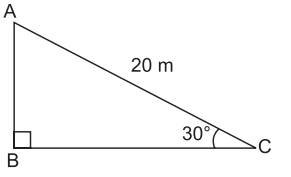
(4) In the given figure, the angle of elevation of the top of a tower AC from a point B on the ground is 60°. If the height of the tower is 20 m, find the distance of the point from the foot of the tower.



(5) A tower stands vertically on the ground. From a point on the ground, which is 15m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

II. Short Answer Type Questions - I [2 Marks]

4. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure given alongside). [NCERT]

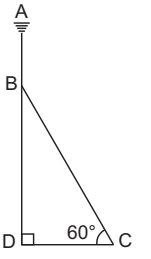


5. From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are 45° and 60° respectively. Find the height of the tower.

[Take $\sqrt{3}$ = 1.73] [AI 2015]

6. The shadow of a tower standing on a level ground is found to be 30 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

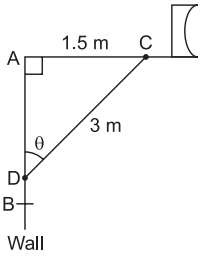
7. An electrician has to repair an electric fault on a pole of height 4 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work (see figure given along side). What should be the length of the ladder that he should use which, when inclined at an angle of 60° to the horizontal would enable him to reach the required position? **[Imp.]**



8. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in the figure. If AC = 1.5 m long and CD = 3 m, find

(*a*) tan θ

(*b*) sec θ + cosec θ [CBSE Standard 2020]



III. Short Answer Type Questions - II [3 Marks]

9. The angle of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}$m high cliff are 45° and 60° respectively. Find the height of the tower. [CBSE 2013]

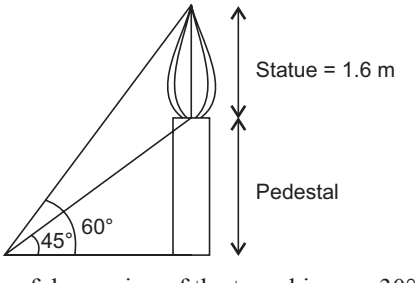
10. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower PQ is 45°. Find the height of the tower PQ and the distance PX. [Use $\sqrt{3}$ = 1.73]

11. From the top of a tower 50 m high, the angle of depression of the top of a pole is 45° and from the foot of the pole, the angle of elevation of the top of the tower is 60°. Find the height of the pole if the pole and tower stand on the same plane. [Foreign 2013]

12. The angle of depression from the top of a tower of a point A on the ground is 30°. On moving a distance of 20 m from the point A towards the foot of the tower to the point B, the angle of elevation of the top of the tower from the point B is 60°. Find the height of the tower and its distance from the point A. [Foreign 2013]

13. A flag-staff stands on the top of a 12 m high tower. From a point on the ground, the angles of elevation of the top and bottom of the flag-staff are observed to be 45° and 30° respectively. Find the height of the flag-staff. **[Imp.]**

14. The given figure shows a statue, 1.6 m tall, standing on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. **[Imp.]**



15. As observed from the top of a 60 m high lighthouse from the sea-level, the angles of depression of the two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. [Use $\sqrt{3}$ = 1.732]

16. The angles of elevation of the top of a tower from two points at a distance of 6 m and 13.5 m from the base of the tower and in the same straight line with it are complementary. Find the height of the tower. [Foreign 2013]

17. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45°. If the height of the lighthouse is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$] [Delhi 2014]

18. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane. [AI 2014]

19. From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flagstaff is fixed at the top of the building and the angle of elevation of the top of the flagstaff from the point P is 45°. Find the length of the flagstaff and the distance of building from the point P. [Delhi 2013]

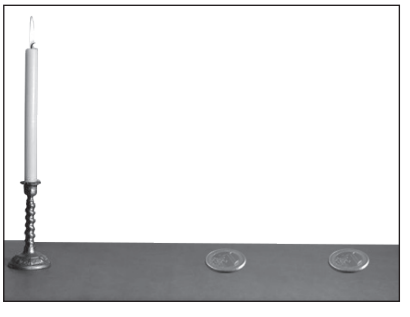
20. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower. [Foreign 2013]

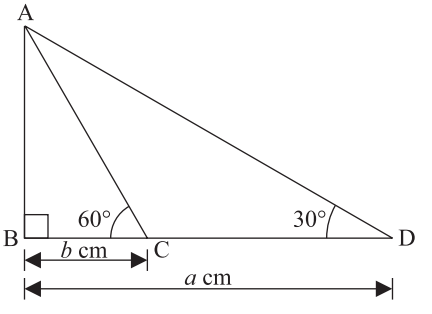
21. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards him. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this, will the car reach the observation tower? [CBSE SP 2018-19] [CBSE 2006 (C)]

22. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$). [Foreign 2015]

23. As observed from the top of a lighthouse, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. [Use $\sqrt{3} = 1.73$] [AI 2016]

24. If the angles of elevation of the top of the candle from two coins distant 'a' cm and 'b' cm ($a > b$) from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.





IV. Long Answer Type Questions [5 Marks]

25. As observed from the top of a 100 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] [CBSE 2018]

26. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. [Use $\sqrt{3} = 1.73$] [Delhi 2015]

27. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct to one place of decimal. [Use $\sqrt{3} = 1.73$] [Foreign 2016]

28. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. [Take $\sqrt{3} = 1.73$] [Foreign 2016]

29. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of the cloud. [CBSE SP 2018-19, 2008, 2010]

30. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is $r \sin\beta \operatorname{cosec} \frac{\alpha}{2}$. [CBSE 2010]

31. A man in a boat rowing away from a lighthouse 100 m high takes 2 minutes to change the angle of elevation of the top of the lighthouse from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$] [Delhi 2019]

32. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles. [Delhi 2019]

33. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. [Given $\sqrt{3} = 1.732$] [Delhi 2019]

34. The angles of elevation and depression of the top and bottom of a lighthouse from the top of a 60 m high building are 30° and 60° respectively. Find

(*i*) the difference between the heights of the lighthouse and the building.

(*ii*) the distance between the lighthouse and the building. [AI 2014]

35. From a point on the ground, the angle of elevation of the top of a tower is observed to be 60°. From a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 30°. Find the height of the tower and its horizontal distance from the point of observation. [AI2016]

36. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 10 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 900 km/hour, find the constant height at which the jet is flying.

[Use $\sqrt{3} = 1.732$] [CBSE 2008]

37. The angle of elevation of the top of a tower as observed from a point on the ground is α and on moving a metres towards the tower, the angle of elevation is β. Prove that the height of the tower is: $\frac{a \tan\alpha \tan\beta}{\tan\beta - \tan\alpha}$.

[NCERT] [CBSE 2006]

38. From a window, x metres high above the ground in a street, the angles of elevation and depression of the top and foot of the other house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $x(1 + \tan\alpha \tan\beta)$ metres. [CBSE 2006]

39. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff.

[Use $\sqrt{3} = 1.73$] [AI 2017]

40. At the foot of a mountain, the angle of elevation of its summit is 45°. After ascending 1 km towards the mountain up an incline of 30°, the elevation changes to 60°. Find the height of the mountain. [Use $\sqrt{3} = 1.732$] [CBSE 2010]

41. The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

[CBSE Standard SP 2020-21, SP 2018]

42. As observed from the top of a 75 m highlight house above the sea level, the angles of depression of two ships are 30° and 45° respectively. If one ship is exactly behind the other on the same side of the lighthouse and in the same straight line, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

43. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

[Use $\sqrt{3} = 1.73$] [CBSE Standard 2020]

44. A verticle tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. [Take $\sqrt{3} = 1.73$] [CBSE Standard 2020]

45. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. [CBSE Standard 2020]

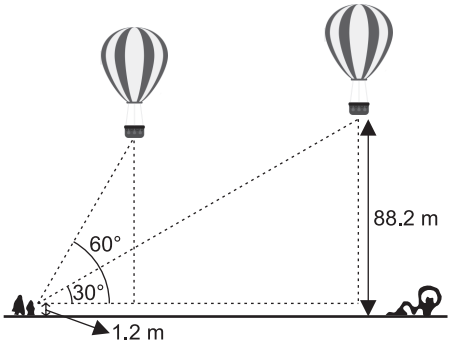
46. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 10 m from the banks, then find the width of the river. [Use $\sqrt{3} = 1.73$]

47. The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

48. The two palm trees are of equal heights and are standing opposite to each other on either side of the river, which is 80 m wide. From a point O between them on the river, the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees. [CBSE Standard SP 2020-21]

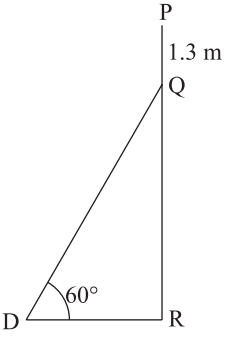
49. The angles of depression of the top and bottom of a 8 m tall building from the top of a multi-storied building are 30° and 45°, respectively. Find the height of the multi-storied building and the distance between the two buildings.

50. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces 30°. Find the distance travelled by the balloon during the interval.

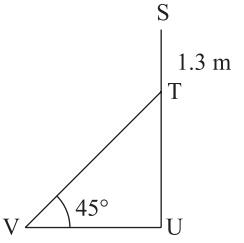


Case Study Based Questions

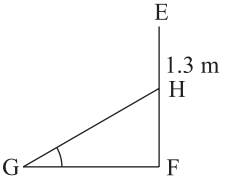
I. Application of Trigonometry—Height of Tree/Tower: Mr. Suresh is an electrician. He receives a call regarding a fault on a pole from three different colonies A, B and C. He reaches one-by-one to each colony to repair that fault.
He needs to reach a point 1.3 m below the top of each pole to undertake the repair work. Observe the following diagrams.



A



B



C

Refer to Diagram A

1. What should be the length of ladder DQ that enable him to reach the required position if the height of the pole is 4 m?

(*a*) $\frac{5\sqrt{3}}{7}$ m (*b*) $\frac{9\sqrt{3}}{5}$ m (*c*) $\frac{7\sqrt{2}}{5}$ m (*d*) $\frac{4\sqrt{3}}{5}$ m

2. What is the distance of the point where the ladder is placed on the ground if the height of pole is 4 m?

(*a*) 2.5 m (*b*) 3.8 m (*c*) 1.56 m (*d*) 5.3 m

Refer to Diagram B

3. Given that the length of ladder is $4\sqrt{2}$ m. What is height of pole?

(*a*) $4\frac{1}{2}$ m (*b*) $4\sqrt{5}$ m (*c*) $5\sqrt{5}$ m (*d*) 5.3 m

4. The distance of the point where the ladder lies on the ground is

(*a*) $3\sqrt{5}$ m (*b*) $4\sqrt{2}$ m (*c*) 4 m (*d*) $4\sqrt{7}$ m

Refer to Diagram C

5. The angle of elevation of reaching point of ladder at pole, *i.e.*, H, if the height of the pole is 8.3 m and the distance GF is $7\sqrt{3}$ m, is

(*a*) 30° (*b*) 60° (*c*) 45° (*d*) None of these.

II. A group of students of class X visited India Gate on an educational trip. The teacher and students had interest in history as well. The teacher narrated that **India Gate**, official name **Delhi Memorial**, originally called **All-India War Memorial**, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

1. What is the angle of elevation if they are standing at a distance of 42 m away from the monument?
(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 0°

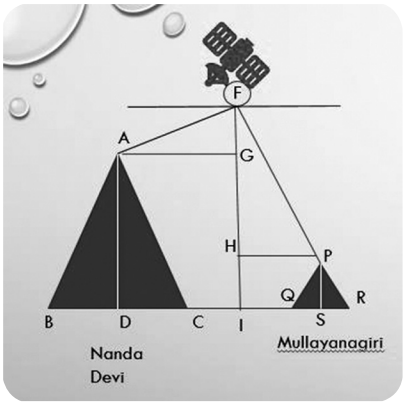
2. They want to see the tower at an angle of 60°. The distance where they should stand will be
(*a*) 25.24 m (*b*) 20.12 m (*c*) 42 m (*d*) 24.25 m

3. If the altitude of the Sun is at 60°, then the height of the vertical tower that will cast a shadow of length 20 m is
(*a*) $20\sqrt{3}$ m (*b*) $\frac{20}{\sqrt{3}}$ m (*c*) $\frac{15}{\sqrt{3}}$ m (*d*) $15\sqrt{3}$ m

4. The ratio of the length of a rod and its shadow is 1:1 . The angle of elevation of the Sun is
(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

5. The angle formed by the line of sight with the horizontal when the object viewed is below the horizontal level is
(*a*) corresponding angle (*b*) angle of elevation (*c*) angle of depression (*d*) complete angle

III. A satellite flying at a height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka, they are being Nanda Devi (height 7,816 m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the distance between the peaks of two mountains is 1937 km, and the satellite is vertically above the mid-point of the distance between the two mountains.



1. The distance of the satellite from the top of Nanda Devi is
(*a*) 1118.29 km (*b*) 577.52 km
(*c*) 1937 km (*d*) 1025.36 km

2. The distance of the satellite from the top of Mullayanagiri is
(*a*) 1139.4 km (*b*) 577.52 km
(*c*) 1937 km (*d*) 1025.36 km

3. The distance of the satellite from the ground is
(*a*) 1139.4 km (*b*) 566.96 km
(*c*) 1937 km (*d*) 1025.36 km

4. What is the angle of elevation if a man is standing at a distance of 7816 m away from Nanda Devi?
(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 0°

5. If a mile stone very far away from, makes 45° to the top of Mullayangiri mountain. So, find the distance of this mile stone from the mountain.
(*a*) 1118.327 km (*b*) 566.976 km (*c*) 1937 km (*d*) 1025.36 km

Answers and Hints

1. (1) (*a*) 60° (1) (2) (*d*) 45° (1)

(3) (*c*) $75\sqrt{3}$ (1) (4) (*a*) 30°, 45° (1)

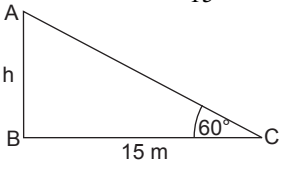
2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)

(2) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) 27 m (1) (2) $10\sqrt{3}$ m (1)

(3) 30° (1) (4) $\frac{20}{\sqrt{3}}$ m (1)

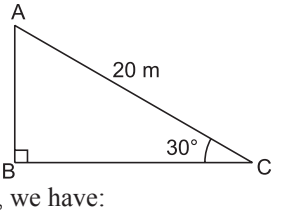
(5) In right ΔABC, $\tan 60° = \frac{h}{15}$



$\Rightarrow \quad \sqrt{3} = \frac{h}{15}$ (½)

$\therefore \quad h = 15\sqrt{3}$ m (½)

4. In the given figure, let AC be the rope and AB be the pole.



In right Δ ABC, we have:

$$\frac{AB}{AC} = \sin 30°$$

$\Rightarrow \quad \frac{AB}{20} = \frac{1}{2}$ [$\because$ AC = 20 m] (1)

$\Rightarrow \quad AB = 20 \times \frac{1}{2} = 10$ m

Thus, the required height of the pole is 10 m. (1)

5. Let AB be 60 m high building and CD be the tower of height *h*.

$\therefore \quad \angle ACE = 45°$ and $\angle ADB = 60°$

(using alternate angles)

Let BD = CE = *x*

BE = CD = *h* $\Rightarrow$ AE = 60 – *h*

In right-angled triangle ABD,

$$\frac{BD}{AB} = \cot 60° \Rightarrow \frac{x}{60} = \frac{1}{\sqrt{3}}$$

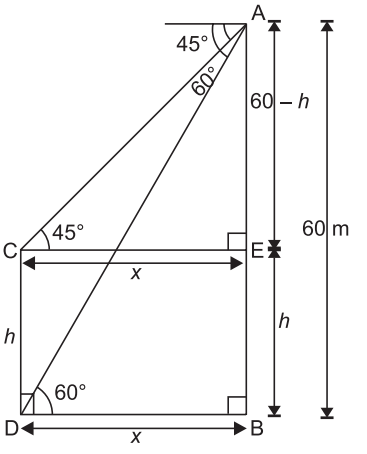
$$x = \frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} \quad ...(i)$$ (1)

In right-angled triangle AEC,

$$\frac{AE}{CE} = \tan 45°$$

$\Rightarrow \quad \frac{60 - h}{x} = 1 \Rightarrow 60 - h = x$

$\Rightarrow \quad h = 60 - 20\sqrt{3}$ [using (*i*)]



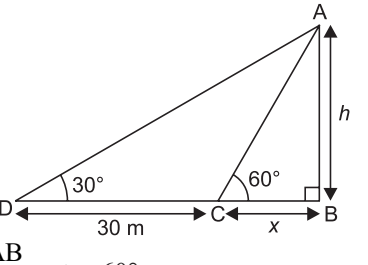
$\Rightarrow \quad h = 20[3 - \sqrt{3}] = 20[3 - 1.73]$

$= 20 \times 1.27 = 25.4$ m

$\therefore$ Height of the tower is 25.4 m. (1)

6. Let AB be the tower of height *h*.

Let BC = *x*



In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(*i*) (½)

In ΔADB, $\frac{AB}{BD} = \tan 30°$

$\Rightarrow \quad \frac{h}{x+30} = \frac{1}{\sqrt{3}} \Rightarrow h\sqrt{3} = x + 30$

$\Rightarrow \quad h\sqrt{3} = \frac{h}{\sqrt{3}} + 30$ [From (*i*)] (½)

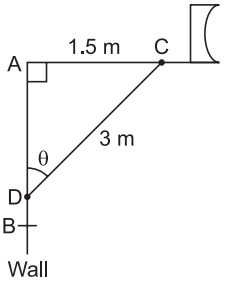
$\Rightarrow \quad h\sqrt{3} = \frac{h + 30\sqrt{3}}{\sqrt{3}} \Rightarrow 3h = h + 30\sqrt{3}$

$\Rightarrow \quad 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3}$

Hence, the height of the tower is $15\sqrt{3}$ m. (1)

7. $\frac{5.4}{\sqrt{3}}$ m (2)

8. ΔACD is a right angled triangle.



So, $CD^2 = AC^2 + AD^2$

$\Rightarrow \quad (3)^2 = (1.5)^2 + AD^2$

$\Rightarrow \quad AD^2 = 9 - 2.25 = 6.75$

$\Rightarrow \quad AD = \sqrt{\frac{6.75 \times 4}{4}} = \sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2}$

(*a*) $\tan\theta = \frac{AC}{AD} = \frac{1.5}{\frac{3\sqrt{3}}{2}} = \frac{1.5 \times 2}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$ (1)

(*b*) $\sec\theta = \frac{CD}{AD} = \frac{3}{\frac{3\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

$\text{cosec}\,\theta = \frac{CD}{AC} = \frac{3}{1.5} = \frac{30}{15} = 2$

$\therefore \quad \sec\theta + \text{cosec}\,\theta = \frac{2}{\sqrt{3}} + 2$

$= \frac{2 + 2\sqrt{3}}{\sqrt{3}}$ (1)

9. In ΔABC,

$\frac{AB}{BC} = \tan 60° \Rightarrow \frac{60\sqrt{3}}{BC} = \sqrt{3}$

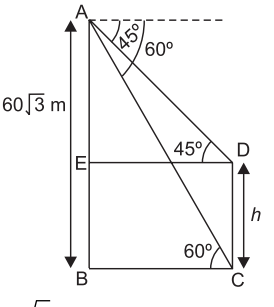
$BC = 60$ m ...(*i*) (1)

Now, in ΔAED,

$\frac{AE}{ED} = \tan 45°$

$\Rightarrow \quad \frac{60\sqrt{3} - h}{BC} = 1 \quad [\because AE = AB - BE \text{ and } ED = BC]$

$\Rightarrow \quad 60\sqrt{3} - h = 60$ [From (*i*)] (1)



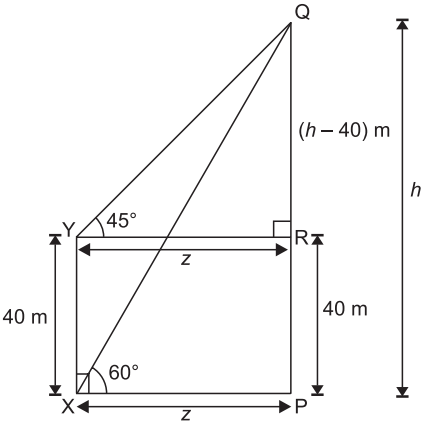
$h = 60\sqrt{3} - 60 \Rightarrow h = 60(\sqrt{3} - 1)$

$h = 60(1.73 - 1)$

$= 60 \times 0.73 = 43.8$ m (1)

10. In ΔQPX, $\frac{PQ}{PX} = \tan 60° \Rightarrow \frac{h}{z} = \sqrt{3}$

$\Rightarrow \quad \frac{h}{\sqrt{3}} = z$ ...(*i*)



In ΔQRY,

$\frac{QR}{YR} = \tan 45° \Rightarrow \frac{h - 40}{z} = 1$

$\Rightarrow \quad h - 40 = z$ ...(*ii*) (1)

From (*i*) and (*ii*), we get

$\frac{h}{\sqrt{3}} = h - 40 \Rightarrow h = h\sqrt{3} - 40\sqrt{3}$

$\Rightarrow \quad h\sqrt{3} - h = 40\sqrt{3} \Rightarrow h = \frac{40\sqrt{3}}{\sqrt{3} - 1}$ (1)

$\Rightarrow \quad h = \frac{40\sqrt{3}(\sqrt{3} + 1)}{2}$

$= 20(3 + \sqrt{3}) = 20(3 + 1.73)$

$\Rightarrow \quad h = 20 \times 4.73 = 94.6$ m ...(*iii*)

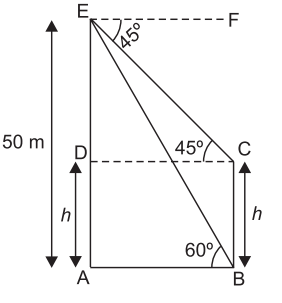
So, height of the tower PQ = 94.6 m and the distance PX = 94.6 – 40 = 54.6 m. [From (*ii*) and (*iii*)] (1)

11. Let the height of the pole is *h*.

In right ΔEDC,

$\tan 45° = \frac{ED}{DC} \Rightarrow 1 = \frac{50 - h}{DC}$

$DC = 50 - h = AB$ ...(*i*) (1)



In right ΔEAB,

$\tan 60° = \frac{EA}{AB} \Rightarrow \sqrt{3} = \frac{50}{AB}$

$\Rightarrow \quad \sqrt{3} = \frac{50}{50 - h}$ [From (*i*)] (1)

$\Rightarrow \quad 50 - h = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow \quad (50 - h) = \frac{50 \times 1.73}{3}$

$\Rightarrow \quad 3(50 - h) = 86.50 \Rightarrow 150 - 3h = 86.50$

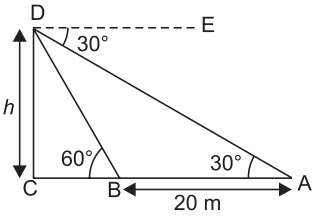
$\Rightarrow 150 - 86.50 = 3h \quad \Rightarrow \quad 63.50 = 3h$

$\Rightarrow \qquad h = \dfrac{63.50}{3} \quad \Rightarrow \quad h = 21.16$ m (1)

12. In right ΔDCB,

$$\tan 60° = \frac{DC}{BC} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{BC}$$

$$\Rightarrow \qquad BC = \frac{h}{\sqrt{3}} \qquad ...(i)\ (1)$$



In right ΔDCA,

$$\tan 30° = \frac{DC}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BC+AB}$$

$$\Rightarrow \quad BC + 20 = \sqrt{3}h$$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} + 20 = \sqrt{3}h \qquad \text{[From } (i)\text{]} \ (1)$$

$$\Rightarrow \quad h\left(\frac{3-1}{\sqrt{3}}\right) = 20 \quad \Rightarrow \quad h = 10\sqrt{3} = 17.30 \text{ m}$$

On putting $h = 10\sqrt{3}$ in equation (i), we get

$$BC = \frac{10\sqrt{3}}{\sqrt{3}} = 10 \text{ m}$$

So, the height of the tower is 17.30 m and its distance from point A = 20 + 10 = 30 m. (1)

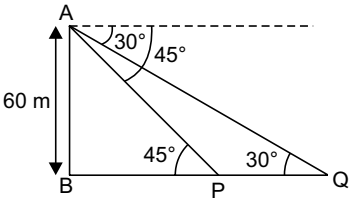
13. $12(\sqrt{3} - 1)$ m (3)

14. $(0.8)(\sqrt{3} + 1)$ m (3)

15. In ΔABP,

$$\tan 45° = \frac{AB}{BP} \quad \Rightarrow \quad 1 = \frac{60}{BP}$$

$$\Rightarrow \qquad BP = 60 \text{ m} \qquad (1)$$



In ΔABQ,

$$\tan 30° = \frac{AB}{BQ} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{BQ}$$

$$\Rightarrow \qquad BQ = AB\sqrt{3} = 60(\sqrt{3})$$

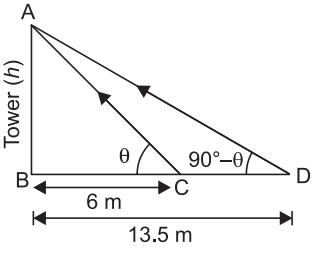
$$= 103.92 \text{ m} \qquad (1)$$

Distance between two ships

$$= BQ - BP = 103.92 - 60$$

$$= 43.92 \text{ m.} \qquad (1)$$

16. In ΔABC, $\tan\theta = \dfrac{h}{6}$ $\qquad ...(i)\ (1)$



In ΔABD,

$$\tan(90° - \theta) = \frac{h}{13.5} \Rightarrow \cot\theta = \frac{h}{13.5}$$

$$\Rightarrow \qquad \tan\theta = \frac{13.5}{h} \qquad ...(ii)\ (1)$$

$$\therefore \qquad \frac{h}{6} = \frac{13.5}{h} \qquad \text{[From } (i) \text{ and } (ii)\text{]}$$

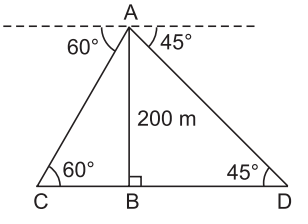
$$h^2 = 13.5 \times 6 = 81.0$$

$$\Rightarrow \qquad h = 9 \text{ m} \qquad (1)$$

17. In ΔABD,

$$\frac{AB}{BD} = \tan 45° \quad \Rightarrow \quad \frac{200}{BD} = 1$$

$$\Rightarrow \qquad BD = 200 \text{ m} \qquad ...(i)\ (1)$$



Now, in ΔABC

$$\frac{AB}{BC} = \tan 60° \quad \Rightarrow \quad \frac{200}{BC} = \sqrt{3}$$

$$\Rightarrow \qquad BC = \frac{200}{\sqrt{3}} \text{ m} \qquad ...(ii)\ (1)$$

$\therefore$ Distance between the ships

$$= BC + BD = \frac{200}{\sqrt{3}} + 200$$

[From (i) and (ii)]

$$= \frac{200\sqrt{3}}{3} + 200$$

$$= \frac{200 \times 1.73}{3} + 200 = \frac{346}{3} + 200$$

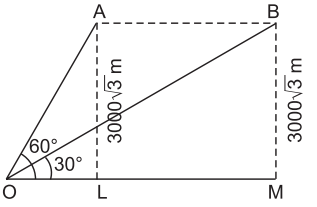
$$= 115.33 + 200 = 315.33 \text{ m} \qquad (1)$$

18. In right-angled triangle OLA,

$$\frac{OL}{AL} = \cot 60°$$

$$\Rightarrow \qquad OL = 3000\sqrt{3} \times \frac{1}{\sqrt{3}}$$

$$= 3000 \text{ m} \qquad ...(i)\ (1)$$



In right-angled triangle OMB,

$$\frac{OM}{BM} = \cot 30°$$

$$\Rightarrow \quad OM = 3000\sqrt{3} \times \sqrt{3} = 9000 \text{ m} \qquad ...(ii)$$

$$\therefore \quad AB = LM = OM - OL$$

$$= (9000 - 3000)$$

$$= 6000 \text{ m} \qquad \text{(From } (i) \text{ and } (ii)\text{) (1)}$$

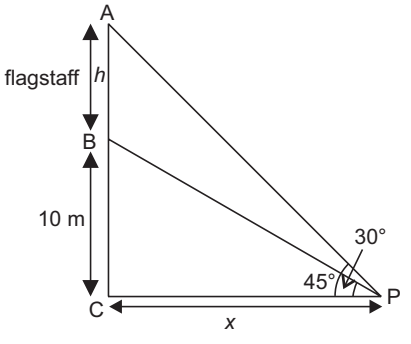
Now, distance covered in 30 s = 6000 m

$\therefore$ Distance covered in 1 hour (3600 s)

$$= \frac{6000}{30} \times \frac{3600}{1000} \text{ km} = 720 \text{ km}$$

$\therefore$ Speed of the aeroplane is 720 km/h. (1)

19. Let height of flagstaff be h and the distance of the building from the point P be x.



In ΔBCP

$$\frac{BC}{CP} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{10}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = 10\sqrt{3} \text{ m} \qquad ...(i) \text{ (1)}$$

In ΔACP, $\frac{AC}{PC} = \tan 45° \Rightarrow \frac{h+10}{x} = 1$

$$\Rightarrow \quad h + 10 = x$$

$$\Rightarrow \quad h + 10 = 10\sqrt{3} \qquad \text{[From } (i)\text{] (1)}$$

$$\Rightarrow \quad h = 10(\sqrt{3} - 1) = 10(1.73 - 1)$$

$$= 10 \times (0.73) = 7.3 \text{ m}$$

$\therefore$ Height of the flagstaff is 7.3 m.

The distance of the building from the point P

$$= 10\sqrt{3} = 10 \times 1.73 = 17.3 \text{ m} \qquad (1)$$

20. In ΔADC,

$$\tan 60° = \frac{CD}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{CD}{AD}$$

$$\Rightarrow \quad AD = \frac{CD}{\sqrt{3}} \qquad ...(i) \text{ (1)}$$

In ΔADE,

$$\tan 30° = \frac{DE}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DE}{AD}$$

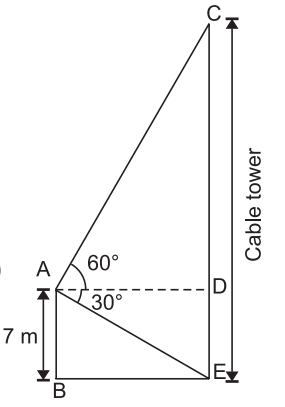
$$\Rightarrow \quad AD = DE\,(\sqrt{3}) \qquad ...(ii) \text{ (1)}$$

From (i) and (ii), we get

$$\frac{CD}{\sqrt{3}} = DE(\sqrt{3})$$

$$\Rightarrow \quad \frac{CD}{\sqrt{3}} = 7(\sqrt{3}) \qquad [\because \ DE = AB = 7 \text{ m}]$$

$$\Rightarrow \quad CD = 21 \text{ m}$$



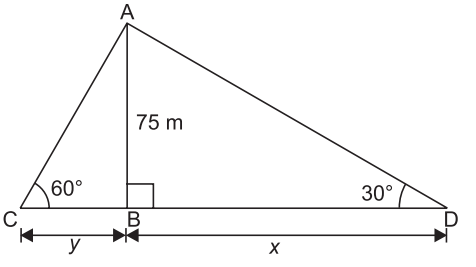
Total height of the cable tower

$$= CD + DE = 21 + 7 = 28 \text{ m} \qquad (1)$$

21. 16.39 mins. (3)

22. Let C and D be the positions of two men.

Let CB = y and BD = x



In ΔABC,

$$\frac{AB}{BC} = \tan 60° \Rightarrow \frac{75}{y} = \sqrt{3} \qquad (1)$$

$$\Rightarrow \quad y = \frac{75}{\sqrt{3}} = \frac{75\sqrt{3}}{3} = 25\sqrt{3} \text{ m} \qquad (1)$$

Now, in ΔABD,

$$\tan 30° = \frac{75}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x}$$

$$\Rightarrow \quad x = 75\sqrt{3}$$

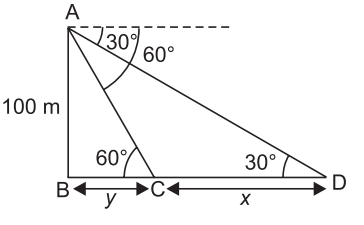
Hence, distance between two men is $x + y$

$$= 25\sqrt{3} + 75\sqrt{3} = 100\sqrt{3}$$

$$= 100 \times 1.732 = 173.20 \text{ m} \qquad (1)$$

23. In ΔABC, $\frac{AB}{BC} = \tan 60°$

$$\Rightarrow \quad \frac{100}{y} = \sqrt{3} \Rightarrow y = \frac{100}{\sqrt{3}} \qquad ...(i) \text{ (1)}$$



In ΔABD,

$$\frac{AB}{BD} = \tan 30° \Rightarrow \frac{100}{y+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x + y = 100\sqrt{3} \Rightarrow x = 100\sqrt{3} - y$$

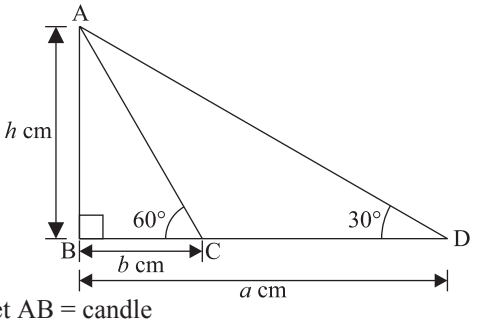
$$\Rightarrow \quad x = 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{300-100}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

[From (*i*)] (1)

$$\Rightarrow \quad x = \frac{200\sqrt{3}}{3} = \frac{200 \times 1.73}{3} = 115.33 \text{ m}$$

The distance travelled by the ship during the period of observation is 115.33 m. (1)

24.



(½)

Let AB = candle

C and D are two coins (½)

$$\tan 60° = \frac{AB}{BC} = \frac{h}{b}$$

$$\sqrt{3} = \frac{h}{b}$$

$$h = b\sqrt{3} \quad ...(i) \text{ (½)}$$

$$\tan 30° = \frac{AB}{BD} = \frac{h}{a}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$h = \frac{a}{\sqrt{3}} \quad ...(ii) \text{ (½)}$$

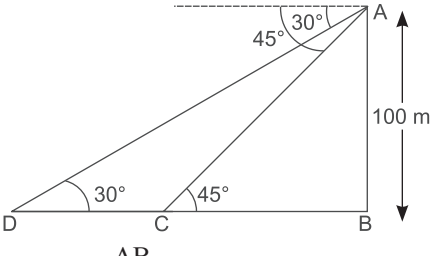
Multiplying (*i*) and (*ii*), we get

$$h^2 = b\sqrt{3} \times \frac{a}{\sqrt{3}} \quad (½)$$

$$h^2 = ba$$

$$h = \sqrt{ab} \text{ m} \quad (½)$$

25. Let AB be the tower and ships are at points C and D.



$$\tan 45° = \frac{AB}{BC} \quad (1)$$

$$\Rightarrow \quad \frac{AB}{BC} = 1 \Rightarrow AB = BC \quad (1)$$

Also $\tan 30° = \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

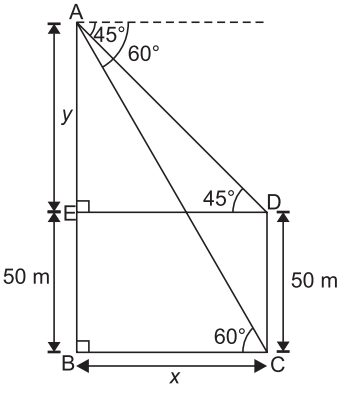
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD} \quad (1)$$

$$\Rightarrow \quad AB + CD = \sqrt{3}\ AB \quad (1)$$

$$\Rightarrow \quad CD = AB(\sqrt{3} - 1)$$

$$= 100 \times (1.732 - 1) = 73.2 \text{ m} \quad (1)$$

26. In ΔAED, $\frac{y}{x} = \tan 45° \Rightarrow \frac{y}{x} = 1$



$$\Rightarrow \quad y = x \quad ...(i) \text{ (1)}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{AE + EB}{BC} = \sqrt{3} \Rightarrow \frac{y+50}{x} = \sqrt{3}$$

$$\Rightarrow \quad x + 50 = \sqrt{3}x \quad [\because\ y = x, \text{ using } (i)] \text{ (1)}$$

$$\Rightarrow \quad \sqrt{3}x - x = 50 \Rightarrow (\sqrt{3} - 1)x = 50$$

$$x = \frac{50}{\sqrt{3}-1} = \frac{50(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{50(1.732+1)}{2} \quad (1)$$

$$x = 68.49 \text{ m} \quad (1)$$

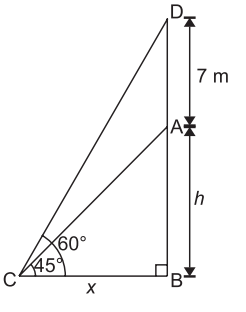
Height of the tower = $50 + y = 50 + 68.49$ $(\because\ x = y)$

$= 118.49$ m

Horizontal distance between the tower and the building $= x = 68.49$ m. (1)

27. In ΔABC, $\frac{AB}{BC} = \tan 45°$

$$\frac{h}{x} = 1 \Rightarrow h = x \quad ...(i) \text{ (1)}$$



Now, in ΔDBC,

$$\frac{DB}{BC} = \tan 60° \Rightarrow \frac{h+7}{x} = \sqrt{3} \quad (1)$$

$\Rightarrow \quad h + 7 = \sqrt{3}h \qquad [\because\ h = x$, using $(i)]$ (1)

$\Rightarrow \quad (\sqrt{3}-1)h = 7 \quad \Rightarrow \quad h = \dfrac{7}{\sqrt{3}-1}$

$= \dfrac{7(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ (1)

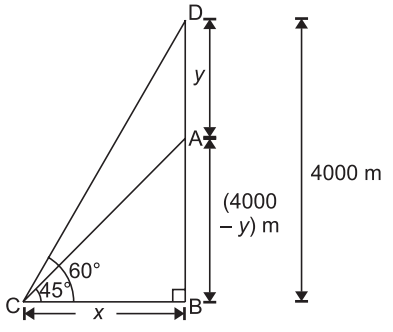
$= \dfrac{7(1.732+1)}{2} = \dfrac{7 \times 2.732}{2}$

$= 9.5$ m

So, height of the tower is 9.5 m. (1)

28. In ΔABC, $\dfrac{AB}{BC} = \tan 45° \quad \Rightarrow \quad \dfrac{4000-y}{x} = 1$

$\Rightarrow \quad x = (4000 - y)$ m ...(i) (1)



Now, in ΔDBC

$\dfrac{DB}{BC} = \tan 60° \quad \Rightarrow \quad \dfrac{4000}{x} = \sqrt{3}$ (1)

$x = \dfrac{4000}{\sqrt{3}}$ ...(ii) (1)

From (i) and (ii),

$4000 - y = \dfrac{4000}{\sqrt{3}} \Rightarrow y = 4000 - \dfrac{4000}{\sqrt{3}}$ (1)

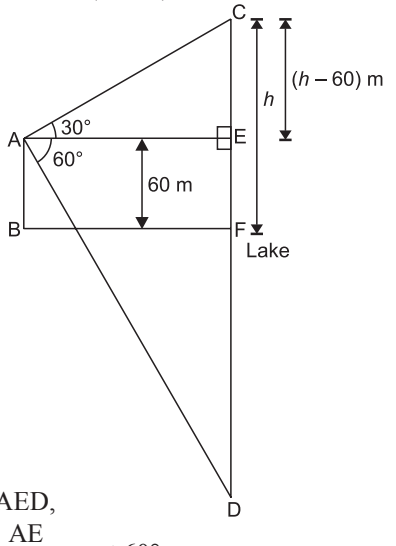
$\Rightarrow \quad y = 4000 - 2312.14 = 1687.86$ m

So, distance between the aeroplanes is 1687.86 m. (1)

29. In right ΔAEC,

$\dfrac{AE}{EC} = \cot 30°$

$\Rightarrow \quad AE = (h-60)\sqrt{3}$ ...(i) (1)



In right ΔAED,

$\dfrac{AE}{ED} = \cot 60°$

$\Rightarrow \quad AE = \dfrac{h+60}{\sqrt{3}}$ ...(ii) (1)

From (i) and (ii), we get

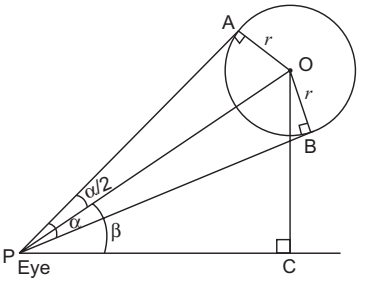
$(h-60)\sqrt{3} = \dfrac{h+60}{\sqrt{3}}$ (1)

$\Rightarrow \quad 3h - 180 = h + 60$ (1)

$\Rightarrow \quad 2h = 240 \quad \Rightarrow \quad h = 120$ m

$\therefore$ Height of the cloud above the lake is 120 m. (1)

30. According to the given statement, the diagram will be as shown:



(1)

In the diagram, O is the centre of the balloon, P is the eye of the observer. If PA and PB are tangents, angle APB = α and $\angle APO = \angle BPO = \dfrac{\alpha}{2}$. (1)

Since the angle of elevation of the centre is β

$\Rightarrow \quad \angle OPC = \beta$

Required to prove: $OC = r \sin\beta \operatorname{cosec} \dfrac{\alpha}{2}$ (1)

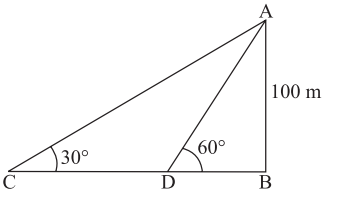
In ΔOAP, $\operatorname{cosec} \dfrac{\alpha}{2} = \dfrac{OP}{OA} \quad \Rightarrow \quad OP = r \operatorname{cosec} \dfrac{\alpha}{2}$.

$(\because\ OA = r)$ (1)

In ΔOPC, $\sin\beta = \dfrac{OC}{OP} \quad \Rightarrow \quad OC = OP \sin\beta$

i.e., $OC = r \operatorname{cosec} \dfrac{\alpha}{2} \sin\beta = r \sin\beta \operatorname{cosec} \dfrac{\alpha}{2}$ (1)

31. Let BD $= x$ m and CD $= y$ m



AB = 100 m

In triangle ABD,

$\tan 60° = \dfrac{100}{x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{100}{x} \quad \Rightarrow \quad x = \dfrac{100}{\sqrt{3}}$ (1)

In triangle ABC,

$\tan 30° = \dfrac{AB}{x+y} \quad \Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{100}{x+y}$

$\Rightarrow \quad x + y = 100\sqrt{3}$ (1)

$\Rightarrow \quad y = 100\sqrt{3} - x = 100\sqrt{3} - \dfrac{100}{\sqrt{3}}$

$\Rightarrow \quad y = \dfrac{200}{\sqrt{3}}$ (1)

$$\text{Now, speed} = \frac{\text{Distance}}{\text{Time}} = \frac{\frac{200}{\sqrt{3}}}{2}$$

$$= \frac{200}{2\sqrt{3}} = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3} \quad (1)$$

$\Rightarrow$ speed of boat

$$= \frac{100 \times 1.732}{3} = \frac{173.2}{3}$$

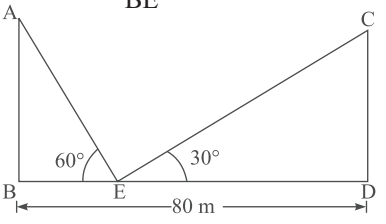
$$= 57.7 \text{ m/min} \quad (1)$$

32. Let AB and CD be the two poles of equal height standing on the opposite sides of the road BD.

$\because$ AB = CD

From figure in right ΔABE

$$\tan 60° = \frac{AB}{BE} \Rightarrow AB = BE\sqrt{3} \quad ...(i)\ (1)$$



(1)

Also, in right ΔCDE,

$$\tan 30° = \frac{CD}{ED} \Rightarrow \frac{AB}{80 - BE} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad AB = \frac{80 - BE}{\sqrt{3}}$$

$$BE = 80 - \sqrt{3}\,AB \quad ...(ii)\ (1)$$

Using (ii) in (i),

$$AB = (80 - \sqrt{3}\,AB)\sqrt{3}$$

$$= 80\sqrt{3} - 3\,AB$$

$$\Rightarrow \quad 4\,AB = 80\sqrt{3} \quad \Rightarrow \quad AB = 20\sqrt{3} \text{ m}$$

$\therefore$ Height of poles $= 20\sqrt{3}$ m (1)

Now, $BE = 80 - \sqrt{3}\ AB$

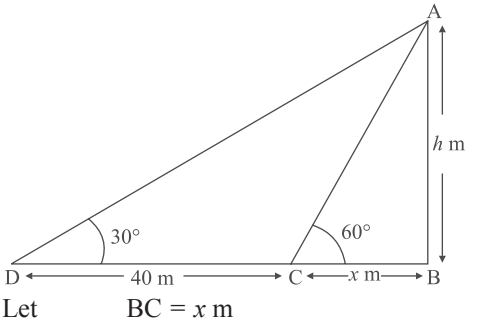
$$= 80 - \sqrt{3} \times 20\sqrt{3}$$

$$= 80 - 60 = 20 \text{ m}$$

$$ED = 80 - BE = 80 - 20 = 60 \text{ m}$$

$\therefore$ Point E is 20 m and 60 m away from both the poles. (1)

33. Let AB be a tower of height h m, and BC its shadow when sun's altitude is 60° and BD also its shadow when sun's altitude is 30°.



(1)

Let $BC = x$ m

Then, $BD = (x + 40)$ m $\quad [\because CD = 40$ m, given]

In right ΔABC,

$$\tan 60° = \frac{AB}{BC} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = x\sqrt{3} \quad ...(i)\ (1)$$

In right ΔABD,

$$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + 40} \quad ...(ii)\ (1)$$

Using (i) in (ii), we get

$$\frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{x + 40} \Rightarrow (x\sqrt{3})\sqrt{3} = x + 40$$

$$\Rightarrow \quad 3x - x = 40$$

$$\Rightarrow \quad 2x = 40 \quad \Rightarrow \quad x = 20 \quad (1)$$

So, $h = 20\sqrt{3}$ [From (i)]

$$\Rightarrow \quad h = 20 \times 1.732 \quad \Rightarrow \quad h = 34.64$$

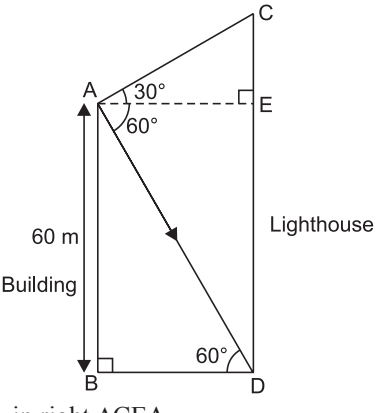
Therefore, the height of the tower is 34.64 m. (1)

34. In right ΔABD,

$$\frac{BD}{AB} = \cot 60° \quad \Rightarrow \quad \frac{BD}{60} = \frac{1}{\sqrt{3}} \quad (1)$$

$$\Rightarrow \quad BD = \frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} \text{ m}$$

$\therefore$ $AE = 20\sqrt{3}$ m $\quad (\because BD = AE)\ (1)$



(1)

Now, in right ΔCEA,

$$\tan 30° = \frac{CE}{AE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{CE}{20\sqrt{3}} \quad \Rightarrow \quad CE = 20 \text{ m} \quad (1)$$

(i) Difference between the heights of the lighthouse and the building = CE = 20 m

(ii) The distance between the lighthouse and the building $= BD = 20\sqrt{3}$ m. (1)

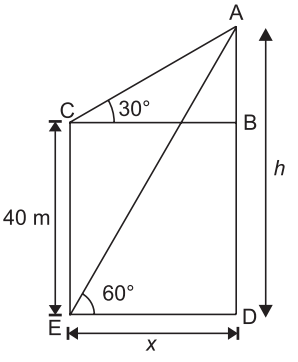
35. Let h be the height of the tower and x be the horizontal distance from the point of observation.

$\therefore$ $CB = ED = x$ and $CE = BD = 40$ m

In right ΔABC,

$$\tan 30° = \frac{AB}{BC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{x}$$

$\Rightarrow \quad x = AB\sqrt{3}$ ...(*i*) (1)



(1)

Now, in right ΔAED

$$\tan 60^\circ = \frac{AD}{DE}$$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(*ii*)(1)

From equations (*i*) and (*ii*), we get

$$AB\sqrt{3} = \frac{h}{\sqrt{3}} \Rightarrow \sqrt{3}(h-40) = \frac{h}{\sqrt{3}}$$

$[\because \ AB + 40 = h \Rightarrow AB = h - 40]$

$3(h-40) = h$

$\Rightarrow \quad 2h = 120 \Rightarrow h = 60$ m (1)

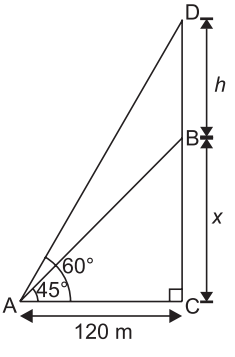
From (*ii*), $x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

$\Rightarrow \quad x = 34.641$ m (1)

36. 2165 m (5)

39. Let BC = x be the tower and BD be the flagstaff of height h.

AC = 120 m, ∠BAC = 45° and ∠DAC = 60°



(1)

In right-angled triangle ACB,

$$\frac{AC}{BC} = \cot 45^\circ \Rightarrow \frac{120}{x} = 1$$

$\Rightarrow \quad x = 120$ ...(*i*) (1)

In right-angled triangle ACD,

$$\frac{CD}{AC} = \tan 60^\circ \Rightarrow \frac{h+x}{120} = \sqrt{3}$$

$\Rightarrow \quad h + x = 120\sqrt{3}$ (1)

$\Rightarrow \quad h = 120\sqrt{3} - 120$ [using (*i*) $x = 120$]

$\Rightarrow \quad h = 120[\sqrt{3} - 1]$ (1)

$\Rightarrow \quad h = 120[1.73 - 1]$ m

$\Rightarrow \quad h = 120 \times 0.73 = 87.6$ m

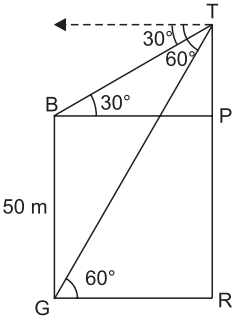
$\therefore$ Height of the flagstaff is 87.6 m. (1)

40. 1.366 km (5)

41. In right ΔBTP,

$$\Rightarrow \quad \tan 30^\circ = \frac{TP}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{TP}{BP}$$

$BP = TP\sqrt{3}$ ...(*i*) (1)



(1)

In right ΔGRT,

$$\tan 60^\circ = \frac{TR}{GR} \Rightarrow \sqrt{3} = \frac{TR}{GR}$$

$\Rightarrow \quad GR = \frac{TR}{\sqrt{3}}$ ...(*ii*) (1)

Now, $TP\sqrt{3} = \frac{TR}{\sqrt{3}}$ $(\because BP = GR)$

$\Rightarrow \quad 3TP = TP + PR \Rightarrow 2TP = BG$

$\Rightarrow \quad TP = \frac{50}{2}$ m = 25 m (1)

Now, $TR = TP + PR = (25 + 50)$ m

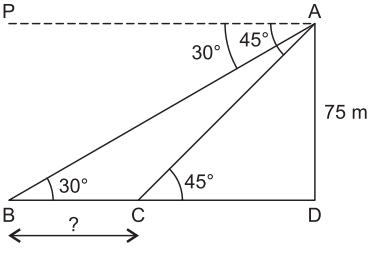
Height of tower = TR = 75 m

Distance between building and tower

$$= GR = \frac{TR}{\sqrt{3}}.$$

$\Rightarrow \quad GR = \frac{75}{\sqrt{3}}$ m $= 25\sqrt{3}$ m. (1)

42.



(1)

In right ΔADC,

$$\tan 45^\circ = \frac{75}{CD} \Rightarrow 1 = \frac{75}{CD}$$

$\Rightarrow \quad CD = 75$ (1½)

In right ΔADB,

$$\tan 30° = \frac{75}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$\Rightarrow$ $BD = 75\sqrt{3}$ (1½)

$\Rightarrow$ Distance between two ships

$= BC = 75(\sqrt{3} - 1)$ m $= 54.9$ m (1)

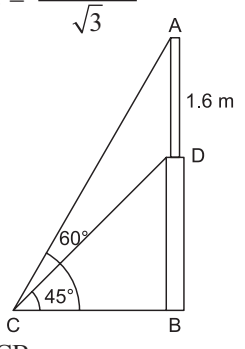
43. Let AD be 1.6 m tall statue, BD the pedestal and C the point of observation such that $\angle ACB = 60°$ and $\angle DCB = 45°$

In right ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AD + BD}{BC} = \frac{1.6 + BD}{BC}$$

$$\Rightarrow \quad BC = \frac{1.6 + BD}{\sqrt{3}} \quad ...(i)\ (1)$$



(1)

In right ΔDCB,

$$\tan 45° = \frac{BD}{BC} \Rightarrow 1 = \frac{BD}{BC}$$

$\Rightarrow$ $BC = BD$ ...(ii) (1)

From (i) and (ii),

$$\frac{1.6 + BD}{\sqrt{3}} = BD$$

$\Rightarrow$ $1.6 + BD = \sqrt{3}\,BD$ (1)

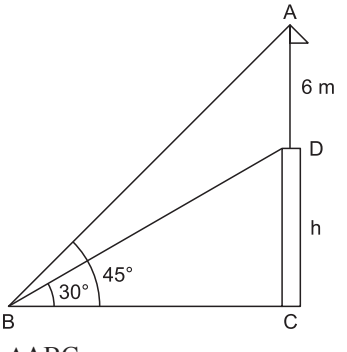
$\Rightarrow \sqrt{3}\,BD - BD = 1.6 \Rightarrow BD(\sqrt{3} - 1) = 1.6$

$$\Rightarrow \quad BD = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{1.6(1.732 + 1)}{2} = 0.8 \times 2.732$$

$= 2.1856 \approx 2.19$ m (1)

44. Let AD be a flagstaff of height 6 m, $CD = h$ be the tower and B be the point of observation.



(1)

In right ΔABC,

$$\tan 45° = \frac{AC}{BC} \Rightarrow 1 = \frac{AD + CD}{BC}$$

$\Rightarrow$ $BC = 6 + h$ ...(i) (1)

In right ΔBCD,

$$\tan 30° = \frac{CD}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BC}$$

$\Rightarrow$ $BC = h\sqrt{3}$ ...(ii) (1)

From equations (i) and (ii)

$$6 + h = h\sqrt{3} \Rightarrow h\sqrt{3} - h = 6$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 6 \Rightarrow h = \frac{6}{\sqrt{3} - 1}$$

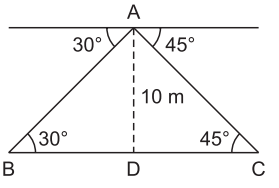
$$\Rightarrow \quad h = \frac{6(\sqrt{3} + 1)}{3 - 1} = 3(1.732 + 1) \quad (1)$$

$\Rightarrow$ $h = 3 \times 2.732 \Rightarrow h = 8.196$

$\Rightarrow$ $h = 8.2$ m (1)

45. 14.64 m (5)

46. Let BC be the river and AD = 10 m be the height of bridge,



(1)

In right ΔABD,

$$\tan 30° = \frac{AD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{BD}$$

$\Rightarrow$ $BD = 10\sqrt{3}$ m ...(i) (1)

In right ΔADC,

$$\tan 45° = \frac{AD}{CD} \Rightarrow 1 = \frac{10}{CD}$$

$\Rightarrow$ $CD = 10$ m ...(ii) (1)

From (i) and (ii),

$BC = BD + CD = 10\sqrt{3} + 10$ (1)

$= 10 \times 1.732 + 10$

$= 17.32 + 10 = 27.32$ m

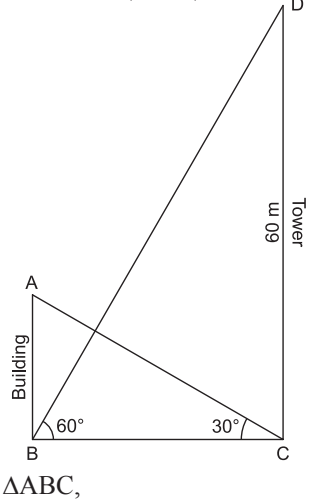
So, the width of the river is 27.32 m. (1)

47. Let AB be the building and CD be the tower of height 60 m.

In right ΔBCD,

$$\tan 60° = \frac{CD}{BC} \Rightarrow \sqrt{3} = \frac{60}{BC}$$

$$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} \text{ m} \quad ...(i)\ (1)$$



(1)

In right ΔABC,

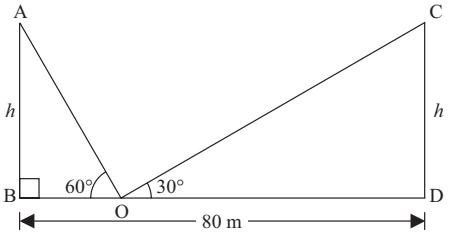
$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC} \quad (1)$$

$$\Rightarrow \quad AB = \frac{BC}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} \quad \text{[From } (i)\text{]} \ (1)$$

$$\Rightarrow \quad AB = 20 \text{ m}$$

So, the height of building = 20 m (1)

48.



(1)

Let BD = river (½)

AB = CD = palm trees = h

BO = x (½)

OD = $80 - x$

In ΔABO, $\tan 60° = \frac{h}{x}$ (½)

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}\,x \quad ...(i) \ (½)$$

In ΔCDO, $\tan 30° = \frac{h}{(80 - x)}$

$$\frac{1}{\sqrt{3}} = \frac{h}{(80 - x)} \quad ...(ii) \ (½)$$

Solving (i) and (ii), we get

$x = 20$

$h = \sqrt{3}\,x = 34.6$ (½)

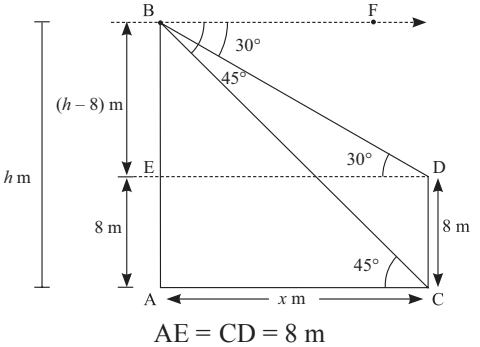
The height of the trees = h = 34.6 m

BO = x = 20 m (½)

DO = $80 - x$

= 80 – 20 = 60 m (½)

49. Let AB be the multi-storied building of height h m and CD the building at a distance x m.



(1)

AE = CD = 8 m [Given]

BE = AB – AE = $(h - 8)$ m

and AC = DE = x m [Given]

Also, $\angle FBD = \angle BDE = 30°$ (Alternate angles)

$\angle FBC = \angle BCA = 45°$ (Alternate angles) (½)

Now,

In right ΔACB,

$$\Rightarrow \quad \tan 45° = \frac{AB}{AC} \quad \left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h \quad ...(i) \ (1)$$

In right ΔBDE,

$$\Rightarrow \quad \tan 30° = \frac{BE}{ED}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h - 8}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(h - 8) \quad ...(ii) \ (1)$$

From (i) and (ii), we get,

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}h - h = 8\sqrt{3}$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 8\sqrt{3}$$

$$\therefore \quad h = \frac{8\sqrt{3}}{\sqrt{3} - 1}$$

$$h = \frac{8\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad (1)$$

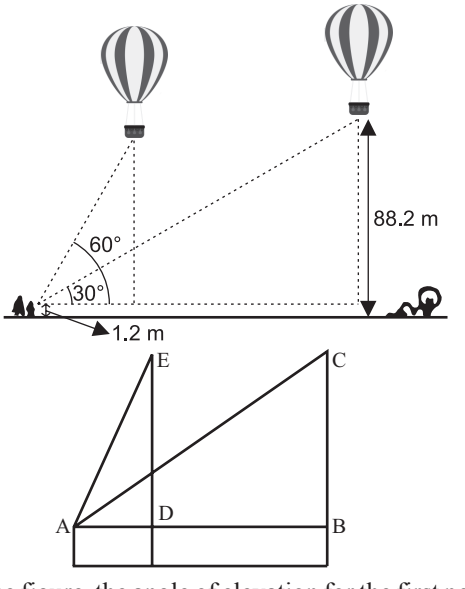
$$h = 4\sqrt{3}(\sqrt{3} + 1)$$

$$h = 12 + 4\sqrt{3} \text{ m}$$

Distance between the two building

$x = (12 + 4\sqrt{3})$ m [From (i)] (½)

50.



From the figure, the angle of elevation for the first position of the balloon $\angle EAD = 60°$ and for second position $\angle BAC = 30°$. The vertical distance

ED = CB = 88.2 – 1.2 = 87 m. (1)

Let AD = x m and AB = y m

Then in right ΔADE, $\tan 60° = \frac{DE}{AD}$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{x}$$

$\therefore \quad x = \dfrac{87}{\sqrt{3}}$ ... (*i*) (1)

In right ΔABC, $\tan 30° = \dfrac{BC}{AB}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{87}{y}$

$\therefore \quad y = 87\sqrt{3}$ ...(*ii*) (1)

Subtracting (*i*) from (*ii*), we get

$$y - x = 87\sqrt{3} - \frac{87}{\sqrt{3}} \quad (1)$$

$$\Rightarrow \quad y - x = \frac{87\sqrt{3} \times \sqrt{3} - 87}{\sqrt{3}} = \frac{87 \times 2}{\sqrt{3}}$$

$$= \frac{87 \times 2 \times \sqrt{3}}{3}$$

$$y - x = 58\sqrt{3} \text{ m}$$

Hence, the distance travelled by the balloon

$$= BD = y - x$$

$$= 58\sqrt{3} \text{ m} \quad (1)$$

Case Study Based Questions

I. **1.** (*b*) $\dfrac{9\sqrt{3}}{5}$ m **2.** (*c*) 1.56 m

3. (*d*) 5.3 m **4.** (*c*) 4 m

5. (*a*) 30°

II. **1.** (*b*) 45° **2.** (*d*) 24.25 m

3. (*a*) $20\sqrt{3}$ m **4.** (*b*) 45°

5. (*c*) angle of depression

III. **1.** (*a*) 1118.29 km **2.** (*c*) 1937 km

3. (*b*) 566.96 km **4.** (*b*) 45°

5. (*c*) 1937 km